Solutions for Exam Physics Laboratory 1: Data and error analysis
25 November 2013

Exercise 1 *(4 points total)*

a) $T = 312.7 \pm 0.3$ K $= (312.7 \pm 0.3) \cdot 10^3$ mK *(1 point)*

b) $\lambda = 1.06 \pm 0.01$ μm $= (1.06 \pm 0.01) \cdot 10^3$ nm *(1 point)*

c) $R = 47.00 \pm 0.04$ kΩ $= (47.00 \pm 0.04) \cdot 10^3$ Ω *(1 point)*

d) $p = 101.30 \pm 0.02$ kPa $= (101.30 \pm 0.02) \cdot 10^3$ Pa *(1 point)*

Exercise 2 *(5 points total)*

a) $v = \sqrt{\frac{5p}{3\rho}} = \sqrt{\frac{5 \cdot 1.01}{3 \cdot 1.21}} = 1.1795$ m/s (intermediate result, keep more decimals). *(1 point)* Note: the speed of sound is very low, because the pressure is very low; at atmospheric pressure $v \approx 300$ m/s.

b) substitute $z = 5p/3\rho$, then $v = \sqrt{z}$ and

$$\left(\frac{\Delta z}{z}\right)^2 = \left(\frac{\Delta p}{p}\right)^2 + \left(\frac{\Delta \rho}{\rho}\right)^2$$

so

$$\frac{\Delta v}{v} = \frac{1}{2}\left(\frac{\Delta z}{z}\right) \Rightarrow \left(\frac{\Delta v}{v}\right)^2 = \left(\frac{1}{2}\right)^2 \left(\frac{\Delta z}{z}\right)^2 = \left(\frac{1}{2}\right)^2 \left[\left(\frac{\Delta p}{p}\right)^2 + \left(\frac{\Delta \rho}{\rho}\right)^2\right]$$

Method of partial derivatives yields same formula (of course). *(2 points for correct formula)*
$\Rightarrow \Delta v/v = 0.009634 \approx 1\% \Rightarrow \Delta v = 0.0114 \approx 0.02$ m/s *(1 point for correct values)*

c) $v = 1.1795 \pm 0.02$ m/s. *(1 point)*

Exercise 3 *(9 points total)*

a)

$$w_1 = \frac{1}{s_1^2} = \frac{1}{0.3^2} = 11.1111 \quad \text{and} \quad w_2 = \frac{1}{s_2^2} = \frac{1}{0.6^2} = 2.7778 \quad (1\ point)$$
$$M = \frac{w_1 M_1 + w_2 M_2}{w_1 + w_2} = \frac{11.1111 \cdot 18.9 + 2.7778 \cdot 19.3}{11.1111 + 2.7778} = 18.98 \text{ kg}$$

(1 point)

b)
$$\frac{1}{s_M^2} = \frac{1}{s_1^2} + \frac{1}{s_2^2} = w_1 + w_2 = 11.1111 + 2.7778 = 13.89$$
$$\Rightarrow s_L = 13.89^{-1/2} = 0.2683 \text{ kg} = \Delta M \quad (2 \textit{ points})$$

In the correct notation: $M \pm \Delta M = 19.0 \pm 0.3$ kg.

c) use standard normal distribution with $z = (x - \mu)/\sigma = (18.8 - 19.0)/0.3 = -0.2/0.3 \approx -0.667 \approx -0.7$, therefore probability $P = \int_{-\infty}^{-0.7} N(y)dy = \int_{0.7}^{\infty} N(y)dy = \frac{1}{2} - \int_0^{0.7} N(y)dy \approx \frac{1}{2} - 0.2580 = 0.2420$ so $P = 24\%$. *(1 point for correct method, 1 point for correct numbers)*

d) $z_1 = (19.1 - 19.0)/0.3 = 0.1/0.3 \approx 0.333 \approx 0.3$ and $z_2 = (19.2 - 19.0)/0.3 = 0.2/0.3 \approx 0.667 \approx 0.7$, therefore probability $P = \int_{z_1}^{z_2} N(y)dy = \int_0^{z_2} N(y)dy - \int_0^{z_1} N(y)dy = 0.2580 - 0.1179 = 0.1401$ so $P = 14\%$. *(2 points for correct method, 1 point for correct numbers)*

Exercise 4 *(6 points total)*

(it is of course the period *of a pendulum, not the resistance)*

a) $\overline{t} = (1/5) \cdot (6.17 + 6.13 + 6.23 + 6.11 + 6.16) = 6.16$ s. *(2 points)*

b) $N = 5; s^2 = \frac{1}{N-1} \sum_{i=1}^{5} (t_i - \overline{t})^2 = \frac{1}{4}((0.01)^2 + (-0.03)^2 + 0.07^2 + (-0.05)^2 + 0^2) = 0.0021 \Rightarrow \sigma = s = \sqrt{0.0021} = 0.0458 \approx 0.05$ s. *(2 points)*

c) $s_m = s/\sqrt{N} = 0.0458/\sqrt{5} = 0.0205 \approx 0.02$ s or ≈ 0.03 s if rounded off upwards. *(2 points)*

Exercise 5 *(12 points total)*

T has the role of x, R the role of y, α the role of a and R_0 the role of b in $y = ax + b$; r = residual.

T(°C)	R (Ω)	r (Ω)
-50.0	80.3	0.1200
-40.0	84.0	-0.1800
-30.0	88.3	0.1200
-20.0	92.0	-0.1800
-10.0	96.3	0.1200

$$a = \frac{N \sum x_i y_i - \sum x_i \sum y_i}{N \sum x_i^2 - \left(\sum x_i\right)^2},$$

$$(\Delta a)^2 = \left(\frac{1}{\sum x_i^2 - N\overline{x}^2} \right) \frac{\sum r_i^2}{N-2},$$

$$(\Delta b)^2 = \left(\frac{1}{N} + \frac{\overline{x}^2}{\sum x_i^2 - N\overline{x}^2} \right) \frac{\sum r_i^2}{N-2}.$$

a) $N = 5, \sum T_i = -150, \sum R_i = 440.9, \sum T_i^2 = 5500, \sum T_i R_i = -12827$
$\Rightarrow \alpha = (5 \cdot -12827 - (-150) \cdot 440.9)/(5 \cdot 5500 - \{-150\}^2) = 0.4\ \Omega/°\mathrm{C}$
and $\overline{T} = -30, \overline{R} = 88.18 \Rightarrow R_0 = \overline{R} - \alpha\overline{T} = 100.18\ \Omega$. *(2 points; $\frac{1}{2}$ point less if no units)*

b) $\sum r_i^2 = 0.108 \Rightarrow (\Delta\alpha)^2 = (5500 - 5 \cdot (-30)^2)^{-1} \cdot (0.108/\{5-2\}) = 3.6 \cdot 10^{-5}$
$\Rightarrow \Delta\alpha = 0.006\ \Omega/°\mathrm{C}$
and $(\Delta R_0)^2 = (1/5 + (-30)^2/\{5500 - 5 \cdot (-30)^2\}) \cdot (0.108/\{5-2\}) = 0.0396$
$\Rightarrow \Delta R_0 = 0.199 \approx 0.2\ \Omega$. *(2 points; $\frac{1}{2}$ point less if no units)*

c) $T = -25.5°\mathrm{C}$ so $R = R_0 + \alpha T = 100.18 + 0.4 \cdot (-25.5) = 89.98\ \Omega$. *(1 point)*
Substitute $z = \alpha T$, then $(\Delta R)^2 = (\Delta R_0)^2 + (\Delta z)^2$ and

$$\left(\frac{\Delta z}{z}\right)^2 = \left(\frac{\Delta\alpha}{\alpha}\right)^2 + \left(\frac{\Delta T}{T}\right)^2 \Rightarrow (\Delta R)^2 = (\Delta R_0)^2 + (\alpha T)^2\left[\left(\frac{\Delta\alpha}{\alpha}\right)^2 + \left(\frac{\Delta T}{T}\right)^2\right] =$$

$$(\Delta R_0)^2 + (T\Delta\alpha)^2 + (\alpha\Delta T)^2 =$$

$$(0.2)^2 + (0.4 \cdot -25.5)^2\left[\left(\frac{0.006}{0.4}\right)^2 + \left(\frac{0.1}{-25.5}\right)^2\right] = 0.0650 \Rightarrow \Delta R = 0.255 \approx 0.3\ \Omega$$

(2 points for correct formula; method of partial derivatives yields same result)
so $R = 90.0 \pm 0.3\ \Omega$ at $T = -25.5°\mathrm{C}$. *($\frac{1}{2}$ point for correct values and $\frac{1}{2}$ point for correct notation)*

d) $\chi^2_{obs} = \sum\{r_i^2/(\Delta R)^2\} = 0.0823/(0.5)^2 = 0.329$. *(2 points)*

e) Three parameters p, q, w are needed for the parabolic fit, so there are $\nu = N - 3 = 5 - 3 = 2$ degrees of freedom. From the table: 10% level $\Rightarrow \chi^2_{table} = 0.211$, 90% level $\Rightarrow \chi^2_{table} = 4.605$. Since χ^2_{obs} is in between those two limits, the parabolic fit is acceptable. *(2 points; if wrong ν but otherwise correct: 1 point less; if wrong values read from tables, but otherwise correct: 1 point less)*

Exam grade = (total of points) / 4 + 1